Basic Logic

1. Simplify each of the following propositions as much as possible (P, Q, R represent propositions):
   (a) $P \vee \text{true}$ (b) $P \vee \text{false}$ (c) $P \wedge \text{true}$ (d) $P \wedge \text{false}$
   (e) $P \vee \neg P$ (f) $P \wedge \neg P$

2. Compare the truth tables or Venn diagrams for the propositions $P \wedge Q$ and P,
   and use this to explain why $(P \wedge Q) \Rightarrow P$. In words, what does this tell us?
   Does $P \Rightarrow (P \wedge Q)$?

3. Complete, and briefly explain using a Venn diagram, the distributive laws for $\vee$ and $\wedge$:
   (a) $P \vee (Q \wedge R) =$
   (b) $P \wedge (Q \vee R) =$

4. Use the fact that $P \Rightarrow Q$ is equivalent to $Q \vee \neg P$, along with the results above, to simplify each of the following as far as possible:
   (a) $(P \Rightarrow Q) \wedge P$
   (b) $(P \Rightarrow Q) \wedge \neg Q$

   In words, what does each of these tell us?

5. Rewrite the following proposition in *twelve* different ways, using only the symbols given:
   $$P \vee (Q \vee R).$$

*6. Using only P, Q, R, $\vee$, and $\wedge$ (along with () as necessary), how many ways can you find to rewrite:
   $$P \wedge (Q \vee R).$$

7. Two of the following propositions are true, and two are false; determine which are which and briefly explain, then express each one in words.
   (a) $\forall\, y \in \mathbb{R}$, $\exists\, x \in \mathbb{R}$ such that $x^3 = y$.
   (b) $\forall\, y \in \mathbb{Z}$, $\exists\, x \in \mathbb{Z}$ such that $x^3 = y$.
   (c) $\exists\, b \in \mathbb{R}$ such that $\forall\, a \in \mathbb{R}$, $b > a$.
   (d) $\forall\, a \in \mathbb{Z}$, $\exists\, b \in \mathbb{Z}$ such that $b > a$.

8. Use the logical laws of negation to pass the $\neg$ symbol in each proposition below past every other logical connective and quantifier:
   (a) $\neg\big[Q \vee (P \wedge \neg R)\big]$.
   (b) $\neg\big[\forall\, M, \exists\, N \text{ such that } \forall\, x, \ (x > N \Rightarrow 2x > M)\big]$.

Logic and sets

9. Which two of our logical connectives and/or constants don't have analogues in the world of sets?
   Briefly, why not?

10. Use the logical definitions of our set operations to unravel each of the following into propositions with *no set operations* other than $\in$; simplify the resulting propositions, if possible:
    (a) $a \in A \cup (B \setminus C)$ (b) $y \in K \triangle (M \cap N)$ (c) $x \in (X \setminus Y) \cup (X \cap Y)$
    (d) $(A \cup B) \subset C$ (e) $(A \setminus C) = C \cap B$